Promoting Multiplication Fact Fluency Through Focus on Relationships

This story is a part of the series:

What's Next? **Stories of Teachers Engaging in Collaborative Inquiry Focused on Using Student Thinking to Inform Instructional Decisions**

What's Next?

Stories of teachers engaging in collaborative inquiry focused on using student thinking to inform instructional decisions

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Introduction

A group of teachers tried to help students to increase their multiplication fact fluency by engaging them in class discussion of contextualized multiplication problems with related numbers. Analysis of a class of fifth-grade students' approaches to solving multiplication fact problems revealed that only a few students used related, known facts to solve problems involving unknown (to them) facts. Consequently, a lesson for the fifth-grade class involving discussion of related multiplication word problems was created and implemented with the intent to help students to leverage relations among known and unknown multiplication facts.

Relevant Florida Mathematics Standards

MAFS.3.OA.2.5 Apply properties of operations as strategies to multiply and divide. Examples: If $6 \times 4 = 24$ is known, then $4 \times 6 = 24$ is also known. (Commutative property of multiplication.) $3 \times 5 \times 2$ can be found by $3 \times 5 = 15$, then $15 \times 2 = 30$. (Associative property of multiplication.) Knowing that $8 \times 5 = 40$ and $8 \times 2 = 16$, one can find 8×7 as $8 \times (5 + 2) = (8 \times 5) + (8 \times 2) = 40 + 16 = 56$. (Distributive property.)

Background Information

Consider reading chapter four in *Children's Mathematics: Cognitively Guided Instruction* (Carpenter et al., 2015). This chapter provides background on the varied ways children solve multiplication and division problems, including the use of known and derived fact strategies. It also expands upon the strategies explained in the Analyzing Student Thinking section.

Another discussion of multiplication fact fluency appears in "Three steps to mastering multiplication facts" (Kling & Bay-Williams, 2015). For more information on assessing fact fluency, consider reading Kling & Bay-Williams (2014), which provides a variety of ways to assess students' fluency with basic math facts.

Carpenter, T. P., Fennema, E., Franke, M. L., Levi, L., & Empson, S. B. (2015). *Children's Mathematics: Cognitively Guided Instruction (2nd. Ed.)*. Portsmouth, NH: Heinemann.

Carpenter, T. P., Franke, M. L., & Levi, L. (2003). *Thinking Mathematically: Integrating Arithmetic and Algebra in Elementary School*. Portsmouth, NH: Heinemann.

Kling, G. (2011). Fluency with basic addition. *Teaching Children Mathematics, 18*(2), 80–88.

Analyzing Student Thinking

In the context of a professional-development experience, a group of teachers conducted short, one-on-one interviews with each of the students in a fifth-grade class. The purpose of the interview was to gain insight into the fifth-graders' fluency with multiplication facts. The interviews involved verbally posing a series of multiplication problems one at a time. If the strategy used by the student to arrive at the answer to each problem was not completely clear, the teacher interviewer asked the student to explain. The teacher interviewers had the option to skip any problem(s) they judged too challenging for the particular student. During the interview, the students had access to manipulatives (linking cubes, fingers) and pencil and paper, but they were not required to use them.

Set of Fact Fluency Items

Students were asked to evaluate each of the expressions in Figure 1, starting with the one at the top of the left column and proceeding down the column, then starting at the top of the right column and proceeding down the column.

As students responded to each item, the teacher interviewer made note of the details of the strategy used (e.g., for 3×5, the student said "5, 10, 15" extending a finger for each count). After the interview, the teachers reflected on the students' strategies using various named categories: *direct modeling*, *skip counting*, *adding*, *repeated doubling*, *fact recall—derived facts*, and *fact recall—*

known facts.[1] These strategies are explained in more detail in the following sections.

Named Strategies Commonly Used to Solve Single-digit Addition Problems

A student using a *direct modeling* strategy to solve the problem represents each and every unit in the problem with some sort of object (e.g., manipulatives, fingers, drawings). For example, when evaluating 7 × 6, a student who uses a *direct modeling* strategy will create seven sets of six objects or six sets of seven objects and determine the answer by counting all of the objects.

A student using a *skip counting* strategy determines the product using a *skip counting* sequence. The student skip counts multiples of one factor while keeping track of the number of counts and stopping at the number of counts specified by the other factor. For example, when solving 7 × 6, a student using *skip counting* might use a count-by-six sequence, "6, 12, 18, 24, 30, 36, 42," extending a finger for each count and recognizing that the product (42) is the number said when the seventh finger is extended.

A student using a *repeated addition* strategy adds multiple sets of one factor the number of times specified by the other factor. For example, when solving 7 × 6, a student using repeated addition might first write down 7 + 7 + 7 + 7 + 7 + 7, making sure to write the numeral "7" six times. Then, he might think, "7 + 7 = 14, 14 + 7 = 21, 21 + 7 = 28" and so on, until all of the sevens have been added.

Some students use *adding* in other ways, such as a *repeated doubling* strategy. Rather than repeatedly *adding* sevens to find the sum of 7 + 7 + 7 + 7 + 7 + 7, a student using doubling recognizes that pairs of sevens can be combined to make fourteens. So, 7 + 7 + 7 + 7 + 7 + 7 is figured out by adding 14 + 14 + 14.

3 × 5	6 × 6
7 × 2	5 × 7
6 × 5	6 × 8
4 × 4	7 × 6
3 × 6	9 × 8
8 × 5	8 × 6
5 × 6	7 × 8
4 × 3	4 × 8
4 × 9	6 × 9

Figure 1. Set of items used in the fact-fluency interview.

A student using a *fact recall—derived facts* strategy uses a known, related fact to help solve a problem involving an unknown fact. For example, to find the unknown product of 7 × 6, a student can use the related fact 7 × 3 = 21. To do so, the student must recognize that seven groups of six (7 × 6) can be decomposed into seven groups of three and seven more groups of three. Drawing on the distributive property of multiplication over addition, the student might think 7 × 6 = (7 × 3) + (7 × 3) = 21 + 21 = 42. Alternatively, a student might think of this same decomposition in a way that is more in line with the associative property: 7 × 6 = 7 × (3 × 2) = (7 × 3) × 2 = 21 × 2 = 42. In either case, the student is using the known fact 7 × 3 = 21 to derive the unknown fact 7 × 6 = 42, usually on the basis of some level of understanding of the fundamental laws, or properties, of arithmetic operations.

A *fact recall—known fact* strategy involves the student's recalling the relevant fact directly from memory. Answers are typically provided quickly. When asked how they got the answer, students often respond that they "just knew" the answer, or "because six times three just equals 18," or "because I did that one before."

Summary of Strategies Used by Students in This Fifth Grade Class

After the teachers analyzed students' strategies for each multiplication fact problem in relation to the categories, they created a chart summarizing

1 The descriptions of strategies presented here are the current descriptions used by our team, and we consider them fluid, as our understanding of these ideas continues to evolve. For a more detailed discussion of these terms, consider reading Carpenter et al. (2015).

their findings (see Figure 2). They recorded the most prevalent strategy used by every student, recognizing that most students used more than one strategy during the interview.

Through examination of the class data, the teachers observed that *counting* and *adding* strategies were the most prevalent for a significant number of these fifth-grade students. Moreover, only a handful of students used *fact recall—derived fact* strategies during the interview. The students who were classified as most often using *fact recall—known fact* strategies did not tend to draw on *derived fact* strategies for facts that they did not know quickly. Rather, they reverted to using *counting* or *adding* strategies. This result led teachers to conjecture that these students' fact knowledge might not be rooted in knowledge of relationships among facts and properties of operations and equality. Therefore, the following learning goal was established for the class of fifth-grade students:

> *Students will notice relationships among multiplication facts and examine how* fact recall—derived fact *strategies reflecting the distributive property of multiplication over addition can be used to solve multiplication problems.*

Planning for the Lesson

With the intention to increase students' use of *fact recall—derived fact* strategies, a lesson was designed for the fifth-grade class that would engage students in class discussion of different strategies for solving a set of related multiplication word problems. The word problems would be designed to evoke strategies involving the use of known facts to figure out problems with unknown facts (i.e., *fact recall—derived fact* strategies).

With this general lesson framework in mind, the teachers discussed different options for the word problems' context and numbers. The teachers wanted to select a context for the word problems to which students could easily relate and that would stimulate interest. They ultimately decided to focus the word problems on a context involving packs of Pokémon cards. Several students in the fifth grade class collected Pokémon cards, and the teachers conjectured that students would be able to easily visualize the groups (packs) and the items in each group (cards). The teachers also decided to include the names of students from the class in the problems. They intentionally selected students who might benefit from the extra personalization of having the problem be about them. At this point, the teachers drafted the following two related word problems, in which the answer to Problem B required completion of Problem A:

A. *Geo is starting a collection of Pokémon cards. He has ___ packs of Pokémon cards with ___ cards in each pack. How many cards does he have?*

Direct modeling	*Counting or adding (e.g., skip counting, repeated addition)*	*Fact recall—derived fact*	*Fact recall—known fact*
Geo	Geluiia	Shawn	Clayton
Geraldine	Emmanuel	Joselyn	Omar
	Johanna	Ayla	Andreya
	Javier	Belinda	Khallani
	Mason	Cheyenne	
	Gabrielle		
	DaSean		

Figure 2. Classification of the students by the most prevalent strategies observed in the interview.

B. *Geo has a good friend named Emmanuel who gives him ___ more packs of Pokémon cards with ___ cards in each pack. How many Pokémon cards does Geo have now?*

Next, the teachers discussed the optimal numbers to place in these word problems in order to stimulate student use of *fact recall—derived fact* strategies reflecting the distributive property of multiplication over addition. The teachers ultimately settled on asking students to find the total number of cards in four packs of eight cards (4 × 8) for Problem A. Eight was selected as a factor, because multiplication tasks involving eight were identified as difficult for many students. Four was selected as the number of packs with the hope that someone would use 2 × 8 to figure out 4 × 8. Teachers also considered that 4 × 8 would serve as a good warm-up problem, because it was one that all students could figure out pretty quickly—even those whose primary strategy was *direct modeling*. For Problem B, the teachers decided to have Emmanuel give Geo eight packs of eight cards. These numbers were selected in hopes that students might notice that they could use the fact 4 × 8 = 32 (from Problem A) to derive the problem 8 × 8 for Problem B.

Although this lesson appeared to open students' minds to the possibility of using fact recall—derived fact *strategies reflecting the distributive property of multiplication over addition, the class will benefit from additional experiences that promote examination of different ways to decompose a given unknown multiplication fact.*

They also conjectured that some students would conceptualize Problem B as a 12 × 8 situation. In addition to being able to examine the decomposition of 12 × 8 suggested by the problem structure [i.e., 12 × 8 = (4 × 8) + (8 × 8)], students might choose to break apart the 12 × 8 in other ways [e.g., 12 × 8 = (10 × 8) + (2 × 8)]. The teachers agreed that class discussion should first involve the class in examining the varied ways that students conceptualized 12 × 8 in their personal solutions, with focus on different ways of decomposing 12 × 8. Then, as needed, the discussion could shift to a focus on finding additional ways to decompose 12 × 8.

Strategy for differentiation to meet the needs of all students in the class

This lesson was developed to be a whole-group discussion in which the class would analyze and discuss student-generated strategies for two related multiplication word problems. The structure of this activity is designed to enable individual students to access the ideas at their own level of understanding. Although the primary lesson focus was on *fact recall—derived fact* strategies, the teachers discussed the importance of helping students using *direct modeling* and *counting* and *adding* strategies to understand how their strategies were related to more sophisticated strategies. They decided therefore to make a deliberate effort to help students using *direct modeling* make sense of *counting* and *adding* strategies. Moreover, the *counting* and *adding* strategies would be intentionally discussed in relation to *fact recall—derived fact* strategies. For students who had a memorized knowledge of facts without an understanding of relationships among facts, the teachers conjectured that explicit discussion of the reasoning behind a range of strategies would be beneficial.

In addition, the teachers discussed how to ensure that the lesson would advance the thinking of students already using *fact recall—derived fact* strategies. First, the teachers planned to encourage students who solved the focal problems very quickly to think about alternative solutions. Along with the rest of the class, these students would be encouraged to explain and justify their own strategies as well as strategies used by others. They would also be expected to articulate similarities and differences among the various strategies shared.

Notes on what to notice about student thinking

During the lesson, the teachers primarily wanted to determine whether students who used *counting* and *adding* strategies in the interview could explain the *fact recall—derived fact* strategies of others and whether they could generate such strategies themselves. For students already using *fact recall—derived fact* strategies, the teachers wanted to determine whether the children could relate their strategies to those of others and whether they could demonstrate flexible thinking by generating multiple *derived fact* strategies for Problem B.

Lesson Plan

This lesson was developed on the basis of the goal set in the *Analyzing Student Thinking* section:

> *Students will notice relationships among multiplication facts and examine how* fact recall—derived fact *strategies reflecting the distributive property of multiplication over addition can be used to solve multiplication problems.*

The structure of this lesson follows the launch, work time, discussion cycle; this cycle is employed for one word problem and then again for a second, related problem. The basic cycle involves the teacher's introducing a problem, allowing time for students to work on the problem on their own, individually, and facilitating student discussion of strategies. The problems of focus in this lesson are as follows:

A. *Geo is starting a collection of Pokémon cards. He has 4 packs of Pokémon cards with 8 cards in each pack. How many cards does he have?*

B. *Geo has a good friend named Emmanuel who gives him 8 more packs of Pokémon cards with 8 cards in each pack. How many Pokémon cards does Geo have now?*

1. Preview lesson plan and establish expectations. Say: "Today we are going to work together to examine different ways to solve a few math problems. You will be expected to solve each problem in your own way and to create a record of your thinking on paper—you might draw pictures, write equations, or explain your thinking with words. Then, as we discuss different ideas as a whole class, your job will be to work hard to understand different ways your classmates have solved each problem and to think about how you can use those strategies in the future."

2. Stimulate interest in and shared background knowledge of the Pokémon card context that

is used for this set of problems.

a. Invite students to raise their hands if they collect Pokémon cards. Tell students that the problems today are about their classmate Geo and his collection of Pokémon cards.

b. Ask students to share how many Pokémon cards come in different kinds of packs, and tell students that the problems they will be solving involve Pokémon cards that come in packs of eight.

3. Launch Problem A: Geo is starting a collection of Pokémon cards. He has 4 packs of Pokémon cards with 8 cards in each pack. How many cards does he have?

   a. Read the problem aloud.

   b. Have students recall the important information in the problem and share with a partner or the whole class.

   c. When you believe that students understand the problem, direct them to figure out the answer to Problem A in a way that makes sense to them. Also, direct students to create a record of their strategy on paper.

   d. Consider posting a written version of the problem at this point.

4. Provide student work time for Problem A, in which each student devises a personal solution and makes a record of that solution on paper.

   a. Circulate and observe students' ways of approaching the problem. Interact with students to ask questions about the details of their mathematical strategies.

   b. Consider pairing early finishers to share and compare strategies. Prompt the students to determine how their strategies are the same and different.

   c. Identify and sequence three to four student solutions to focus on in class discussion, and determine what you want students to learn from the examination of each solution.

      i. Consider sequencing solutions selected for focus from less to more sophisticated strategies (e.g., *direct modeling*, *counting* or *adding*, *fact recall—derived facts*).

      ii. Consider how discussion of each student solution can be used to move students closer to achieving the lesson goal.

      iii. Be prepared to provide students with guidance and assistance in using conventionally accepted written notation to express their ideas.

5. Facilitate class discussion of students' strategies for Problem A.

   a. Pull students attention to the front of the room, and explain that next you have selected a few students' ways of solving the problem that you want the whole class to examine and discuss.

   b. Prompt students to explain and analyze each strategy in the preestablished sequence.

      i. When a student explains his or her mathematical strategy, prompt the other students to reexplain the steps of the strategy and justify why they make sense. Ask: What did [your classmate] do first? Why did [your classmate] do that?

      ii. Consider sometimes presenting student work on the document camera and having the class make conjectures about what the student did. Then the student whose work is being shared can verify or refute the conjectures.

iii. Ask questions to help students "notice" important aspects of each solution. This process can sometimes be supported through questions that prompt comparison of strategies.

6. Repeat steps 3, 4, and 5 for Problem B: *Geo has a good friend named Emmanuel who gives him 8 more packs of Pokémon cards with 8 cards in each pack. How many Pokémon cards does Geo have now?*

   In selecting solutions to focus on in discussion, consider looking for the following:

   a. Solutions that group the eights in different ways with *counting* and *adding* strategies. These solutions can be used to draw connection between additive strategies and multiplicative strategies employing the distributive property. For example, a student might make the addition easier by noticing that every four eights can be thought of as 32, $(8 + 8 + 8 + 8) + (8 + 8 + 8 + 8) + (8 + 8 + 8 + 8) = 32 + 32 + 32$. You might encourage students to consider how this kind of additive solution might be recast to use multiplication, $12 \times 8 = (4 \times 8) + (4 \times 8) + (4 \times 8)$.

   b. Solutions that use *fact recall—derived fact* strategies for determining 12×8. Careful examination and comparison of these solutions can be used to stimulate thinking about how a multiplication problem can be decomposed in different ways (that reflect the distributive property).

      For example:

      1. Students might use the groupings suggested by the problem: $12 \times 8 = (4 \times 8) + (8 \times 8) = 32 + 64 = 96$ cards.

Continued attention must also be given to cultivating students' understanding of how to notate multiplication strategies that reflect the distributive property of multiplication.

    2. Students might think of the 12 packs of cards as ten packs and four packs, 12 × 8 = (10 × 8) + (2 × 8) = 80 + 16 = 96.

7. Close the lesson by asking students to reflect on what they have learned or what they want to remember from today's lesson. Invite students to share their ideas with the class.

Reflection

As anticipated, all of the fifth-grade students were able to make sense of Problem A (involving finding the number of Pokémon card in four packs of eight) fairly quickly. The teacher opened discussion of Problem A by acknowledging that some students "just knew" the answer because, for them, the problem involved a known multiplication fact. Then, she explained that she wanted the class to focus their discussion on different ways of figuring out multiplication problems if you don't "just know." Next, the teacher identified particular students representing a range of strategies to share their solutions for Problem A (see Figure 3 for public record of strategies shared).

After having DaSean explain his *direct modeling* strategy (picture of four rectangles with eight dots in each), the teacher prompted the class to explain how the rectangles represented the four packs of cards and dots represented the Pokémon cards in each pack. Then, as the next two students shared a *skip counting* strategy (8, 16, 24, 32) and a repeated addition strategy (8 + 8 + 8 + 8 = 32), the teacher prompted students who had used *direct modeling* to explain how these *counting* and *adding* strategies related to the *direct modeling* picture. Next, the teacher asked a student who had used a doubling strategy (i.e., 8 + 8 + 8 + 8 = 16 + 16 = 32) to share, and she explained how that student's solution could be thought of in terms of multiplication: (2 × 8) + (2 × 8) = 16 + 16 = 32. She also suggested to students that a really helpful way to think about hard multiplication problems is to think, "How can I use a multiplication fact that I already know to help me with this problem?"

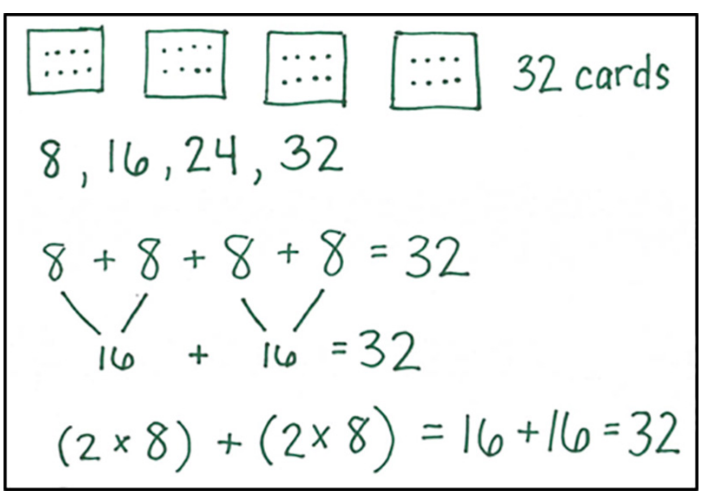

Figure 3. A public record of strategies shared during discussion of Problem A (Geo is starting a collection of Pokémon cards with 8 cards in each pack. How many cards does he have?)

As anticipated, Problem B was more challenging for students to solve than Problem A. Just as in the interviews, the most prevalent approach to the problem was use of *adding* strategies. Although several students created a pictorial representation to support their conceptualization of the problem (for example, see Figure 4a), only two students used a classic *direct modeling* strategy, which involved depicting every individual card. Perhaps stimulated by the discussion of Problem A, several students incorporated doubling in their *adding* strategies (for example, see Figure 4b). Only a few students recorded multiplication notation on their papers for Problem B (for example, see Figure 4d).

The four student work samples examined in class discussion of Problem B are presented in Figure 4. The teacher opened class discussion by displaying Geluiia's strategy (Figure 4a) and having the class make conjectures about how she was thinking. The students identified one row of four packs as the cards Geo had first and the other two rows of four packs each as the cards Emmanuel gave him. The teacher then posed the question, "Why do you think Geluiia organized the cards from Emmanuel in two rows of four packs?" After some opportunity for partners to share their ideas, Geluiia explained that she did that because she already knew 4 × 8 = 32. The teacher reframed Geluiia's explanation saying, "So you thought of the 8 × 8 as 4 × 8 plus 4 × 8? [turning to the class] Can

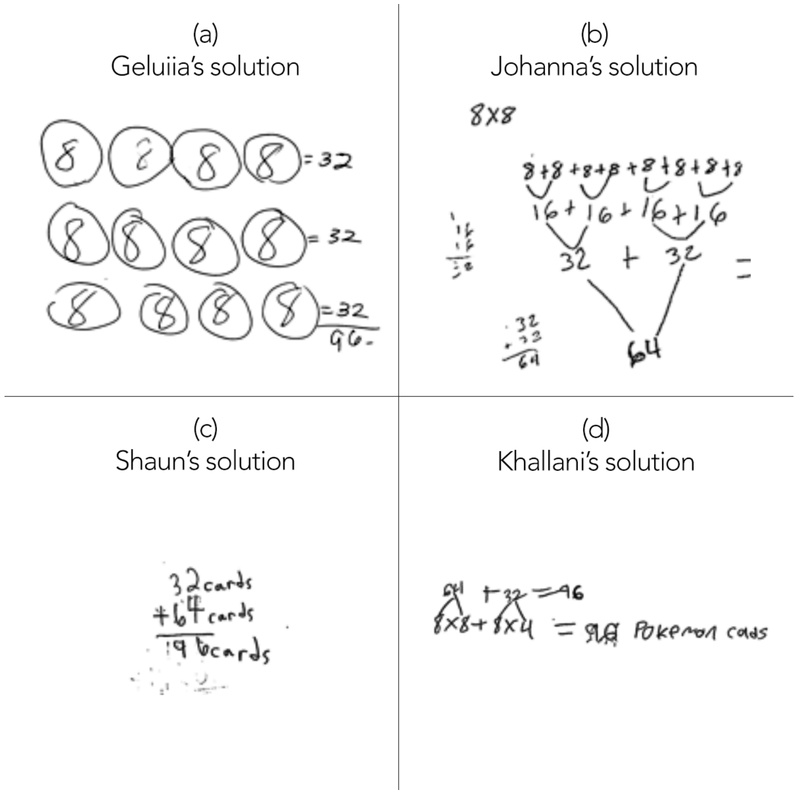

*Figure 4. Student work samples examined in class discussion of Problem B (*Geo has a good friend named Emmanuel who gives him 8 more packs of Pokémon cards with 8 cards in each pack. How many Pokémon cards does Geo have now?)

she do that?" In response, a student explained that she can do that because there are 8 groups in 8 × 8 and (4 × 8) + (4 × 8). Finally, the teacher had the class explain the meaning of the 96 on Geluiia's paper.

Next, the teacher displayed Johanna's solution (Figure 4b) and directed the class to determine what Johanna was thinking. One student noted that Johanna made finding 8 × 8 easier by putting the eights together to make 16s, the 16s together to make 32s, and then the 32s to make 64. Another student noted that Johanna's solution only shows the number of cards that Geo got from Emmanuel and not the total. The teacher then had the students compare Johanna's partial solution with Geluiia's picture and explain what else Johanna needed to do (add 32, for the cards Geo had from Problem A).

Sean's paper (Figure 4c) was displayed next. The teacher asked the class to talk to a partner about where the numbers 32 and 64 in Sean's strategy came from. After a minute or so, multiple partnerships had explained (to each other) that the 32 represented the four packs of eight cards from Problem A, and the 64 represented the eight packs of eight cards from Emmanuel. At this point, the teacher displayed Khallani's paper (see Figure 4d) and guided students to explain Khallani's multiplication notation.

Next, the teacher reported that Clay found his answer by multiplying 12 × 8, and she asked, "Is that okay? I don't see the number twelve anywhere in these problems." Again, partners were directed to briefly talk about this question. A few partnerships were observed to articulate that the 12 represented the total number of packs of Pokémon cards, but other partnerships were unsure. At this point, the teacher guided students to look back at Geluiia's picture and notice the total number of packs represented (12) and the number of cards in each pack (8). Next she guided the class to help her represent Geluiia's strategy using multiplication notation, and she recreated a record of Khallani's strategy. As she made the following record on the board, she prompted students to reexplain the meaning of the numbers in the context of the Pokémon cards:

$$
\begin{aligned}
12 \times 8 &= (4 \times 8) + (4 \times 8) + (4 \times 8) \\
&= 32 + 32 + 32 \\
&= 96
\end{aligned}
$$

$$
\begin{aligned}
12 \times 8 &= (8 \times 8) + (4 \times 8) \\
&= 64 + 32 \\
&= 96
\end{aligned}
$$

Finally, the teacher asked, "Are there other ways we could break up the packs to make is easier to figure out 12 packs of eight?" Only two students raised their hands. After several seconds of wait time, the teacher offered an idea, "How might it work if I count three packs first. Maybe I just know the answer to 3×8. Can I use it for this problem?" With teacher support, the class worked out that $12 \times 8 = (3 \times 8) + (3 \times 8) + (3 \times 8) + (3 \times 8)$. Then they went on to generate two additional ways to decompose 12×8, including $12 \times 8 = (6 \times 8) + (6 \times 8)$ and $12 \times 8 = (10 \times 8) + (2 \times 8)$.

This lesson was effective at increasing student awareness of relationships among multiplication facts. By the end of the lesson, most students were able to explain multiple ways to use "smaller" known facts to find the product of 12×8. The teacher's continual prompting to explain how various solutions were related to the packs of Pokémon cards context seemed especially helpful at guiding students to a conceptual understanding of the varied ways 12×8 might be decomposed. The introduction of a symbolic way to notate *fact recall—derived fact* strategies also appeared helpful and gave the class a common way to communicate their thinking in the future. Examination of a variety of strategies, including those at different levels of sophistication, was also seen as key to helping students with different profiles of understanding follow the lesson to its conclusion.

Although this lesson appeared to open students' minds to the possibility of using *fact recall—derived fact* strategies reflecting the distributive property of multiplication over addition, the class will benefit from additional experiences that promote examination of different ways to decompose a given unknown multiplication fact. Continued attention must also be given to cultivating students' understanding of how to notate multiplication strategies that reflect the distributive property of multiplication. In order to optimize understanding, such notation should be carefully examined in relation to an appropriate (or established) problem context and, when possible, visual models of a given multiplication fact. In addition to lessons focused on related sets of multiplication word problems (like the one featured here), students would probably benefit from focused effort on generating *derived fact* strategies for particular multiplication facts that they personally find difficult to recall.

What's Next?

Stories of teachers engaging in collaborative inquiry focused on using student thinking to inform instructional decisions

What's Next? is a collection of stories documenting professional development experiences shared by elementary teachers working collaboratively to study the complex process of teaching and learning mathematics. Each story in the collection describes practicing teachers studying the thinking processes of real students and using what they learn about those students to make decisions and try to help advance those students' understanding on that day.

The teachers in each story start by learning about how individual students are solving a set of mathematics problems. They use this freshly gathered knowledge of student thinking to develop near-term learning goals for students and a lesson plan tailored to specific students on that specific day. One of the teachers implements the planned lesson while the other teachers observe in real time. The teachers then gather to discuss and reflect on their observations and insights.

In these lessons, the practice of teaching is slowed way down. The stories tell of teachers who are studying student thinking and using that information to plan and implement instructional decisions at a pace that is much slower than it occurs in daily practice. The stories in this collection also depict many aspects in common with formative assessment and lesson study, both of which are a process and not an outcome.

The stories depict real situations that occurred in real time and include both successes and shortcomings. We hope that the stories may be studied and discussed by interested educators so that the lessons and ideas experiences of these teachers and instructional coaches may contribute to additional learning and sharing among other interested teachers.

Learn more about these and other stories at **http://www.teachingisproblemsolving.org/**